Chapter 3

AGS-to-RHIC Transfer Line

The AGS to RHIC (AtR) transfer line[23] has been designed to transport proton beams in the energy range, from 20.58 GeV (just above the RHIC transition Energy) to a maximum injection energy of 28.3 GeV. This range of injection energies is not however applicable for a polarized proton beam which is confined to a shorter range of injection energies. The range of energies of a polarized proton beam to be injected from the AGS to RHIC via the AtR transfer line has been studied before[22], but a more detailed and complete study is the subject of this chapter. The study consists of calculating the stable spin direction of a polarized proton beam from AGS, at the RHIC injection point (defined below), as a function of the proton energy. This energy dependence of the stable spin direction of an injected polarized beam from AGS to RHIC is due to the relative location of the horizontally and vertically bending magnetic elements of the AtR line. In addition a modification of the AtR line is proposed. This modification of the AtR transfer line can increase the energy range that a polarized proton beam can be transferred into RHIC machine with optimum polarization transfer.

3.1 AtR Sections Affecting the Proton Beam Polarization

The layout of the (AtR) transfer line is shown in the schematic diagram of Fig. 3.1. In the AtR transfer line there are two sections of the beam line where the beam direction is not parallel to the horizontal plane but makes an angle below the horizontal plane. The first of the sections is located in the W-line (see Fig. 3.1) within the 20° bend. The second section is located at the end of the Y-line in the RHIC injection section (see Fig. 3.1). A schematic diagram of the first of these two sections is shown in Fig. 3.2 and will be referred to in this chapter as the "12.5 mrad vertical bend."

The "12.5 mrad vertical bend" consists of a -12.51 mrad vertically bending magnet located after the second horizontally bending magnet of the 20° W-line[23] (see also Fig. 3.1) followed by six combined-function dipole magnets which bend the beam horizontally to the right by 15°, and finally a +12.51° mrad vertically bending dipole which restores the beam direction to the horizontal plane. A schematic diagram of the second vertical bend is shown in Fig. 3.3. and will be referred to in this chapter as the "3.0 mrad

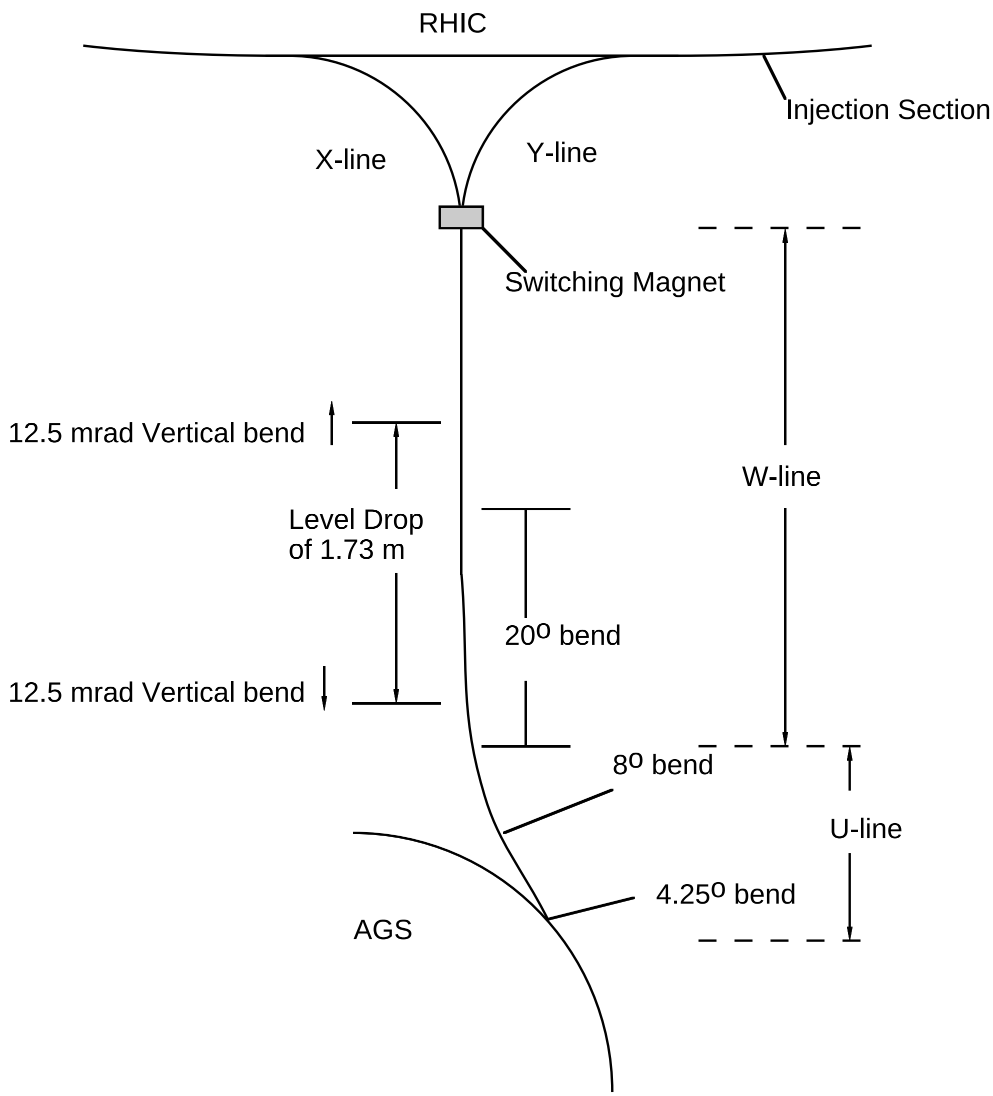

Figure 3.1: Schematic diagram of the AGS to RHIC (AtR) transfer line. The 12.51 mrad vertical bend is located at the region of the 20° horizontal bend. The 3 mrad bend is located at the end of the RHIC injection section shown in the figure.

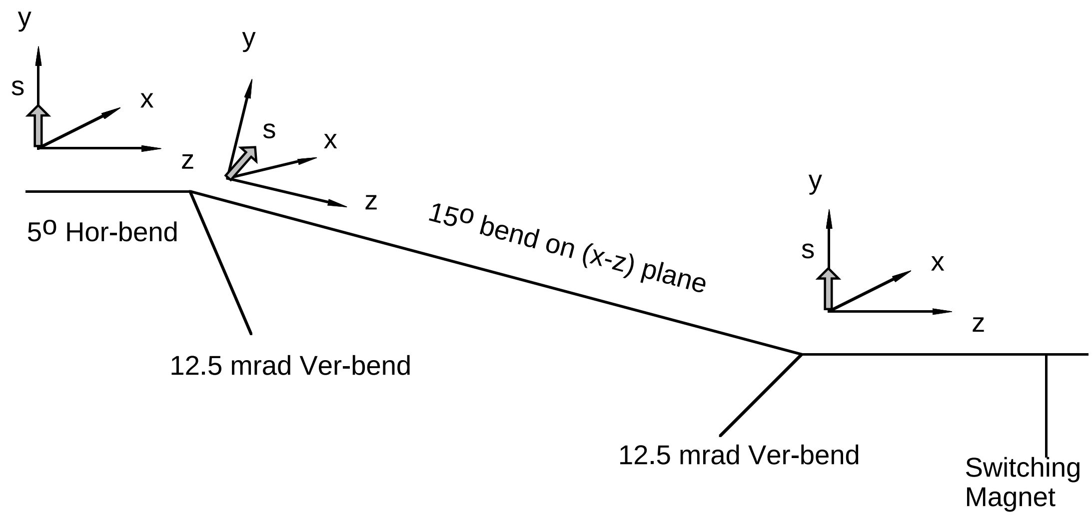

Figure 3.2: Schematic diagram of the "12.5 mrad vertical bend." A −12.51 mrad vertical bend is followed by a 15° horizontal bend and +12.51° vertical bend.

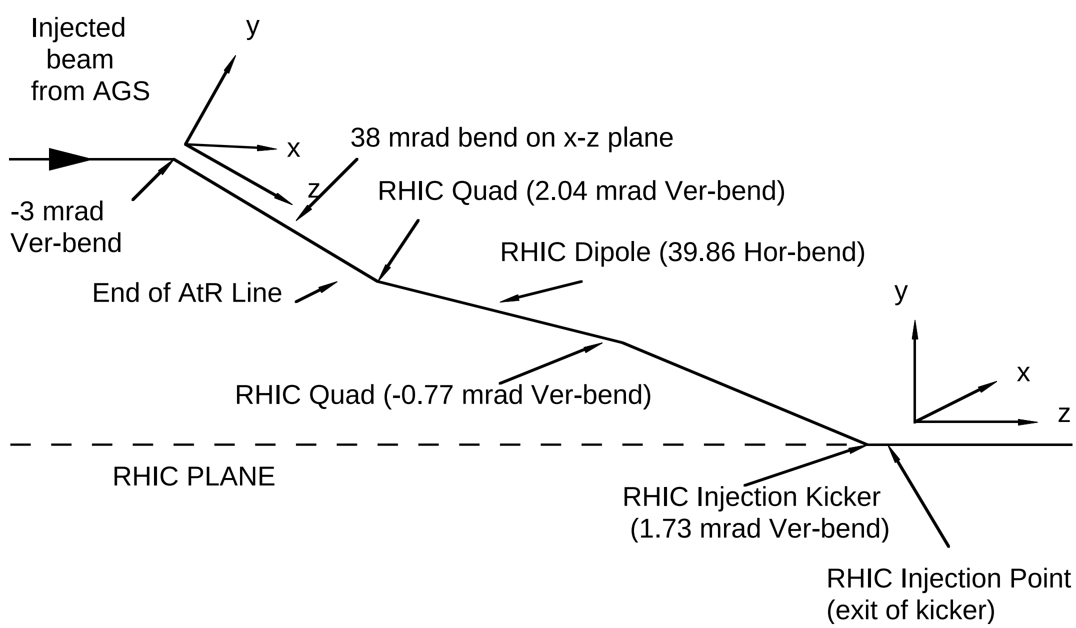

Figure 3.3: Schematic diagram of the "3.0 mrad vertical bend." The beam is injected off axis with respect to the two RHIC quadrupoles which act also as vertical bending magnets. This injection geometry reduces the required strength of the vertical injection kicker, to a value of 1.73 mrad.

vertical bend."

The "3.0 mrad vertical bend" is located at the end of the injection section of the AtR transfer line and begins with a −3.0 mrad vertically bending magnet (see Fig. 3.3) followed by a Lambertson septum magnet[24] which bends the beam 38.0 mrad to the right, then a RHIC quadrupole (focusing) – dipole (left-bend) – quadrupole (defocusing) and finally the vertically bending RHIC injection kicker which deflects the beam by +1.73 mrad which restores the injected beam onto the RHIC plane. The strength required for the vertical injection kicker to restore the beam back to the horizontal plane has been reduced down to 1.73 mrad because part of the vertical bend has been accomplished by the two RHIC quadrupoles. (For the injection into the clockwise beam circulating ring, the polarity of both, the Lambertson injection magnet and RHIC dipole have to be inverted.) The exit point of the RHIC injection kicker is defined here as the "RHIC injection point." It is the arrangement of the bending magnets in these two vertical bends (vertical bend followed by horizontal bend and then vertical) that affects the stable spin direction of a polarized proton beam.

3.2 Calculation of Stable Spin Direction at RHIC Injection Point

The stable spin direction at the exit of the "12.5 mrad vertical bend" has been calculated[22] earlier. In those calculations the initial stable spin direction had been assumed to be vertical and the effect of the "3.0 mrad vertical bend," discussed above, was not included in the calculations since the "3.0 mrad vertical bend" was introduced in the AtR line after the calculations. In the present calculations, the stable spin direction was calculated at the "RHIC injection point" (see Fig. 3.3) and the following items were included in the calculations.

1. *A section of the AGS machine (from the center of the straight section C10 of AGS to the H13 fast extraction point) followed by the AtR line (see Fig. 3.1).* The inclusion of this section of the AGS (C10 to H13) in the calculations makes it possible to take into account the effect of the AGS extraction bumps, the extraction kicker at G10 straight section, and the extraction septum at H10 straight section[25].

2. *The energy dependence of the stable spin direction of a polarized proton beam circulating in the AGS machine.* This energy dependence is due to the partial Snake. For a given strength of the AGS partial Snake, the stable spin direction of a polarized proton beam in the AGS, is well defined anywhere along a closed beam orbit in the AGS. The center of the C10 straight section has been chosen as the starting location of all the required ray trace calculations. This starting location has been chosen because of item 1 above.

3. *The finite beam size and momentum spread of the beam.* In order to calculate the spread of the stable spin direction at the "RHIC injection point," the shape of the beam ellipsoid at the location "C10" of the AGS machine is required. Closed orbit calculations[25] based on measured magnetic field maps of the combined function magnets of the AGS were performed for the AGS machine. From

these calculations the beam parameters at any given point along a closed orbit in AGS machine were calculated, and for a given emittance, the beam ellipsoid at the location C10 of the AGS was calculated.

The calculations to determine the stable spin direction at the "RHIC injection point" were based on the numerical integration of two equations: the differential equation of motion (Eq. 3.1) of a charged particle moving in static magnetic and electric fields, and the spin precession equation (Eq. 3.2) of a particle with magnetic moment in the same electric and magnetic field:

$$\frac{d\mathbf{v}}{dt} = \frac{e}{m\gamma}\left(\mathbf{v}\times\mathbf{B} + \mathbf{E} - \frac{(\mathbf{v}\cdot\mathbf{E})\,\mathbf{v}}{c^2}\right) \tag{3.1}$$

$$\frac{d\mathbf{s}}{dt} = \frac{e}{m\gamma}\left((G\gamma+1)\,\mathbf{s}\times\mathbf{B} + (\gamma-1)\,G\frac{(\mathbf{v}\cdot\mathbf{B})\,(\mathbf{v}\times\mathbf{s})}{v^2} + \left(G\gamma + \frac{1}{1+\frac{1}{\gamma}}\right)\frac{\mathbf{s}\times(\mathbf{E}\times\mathbf{v})}{c^2}\right) \tag{3.2}$$

In (Eq. 3.1) and (Eq. 3.2), above B and E are the magnetic and electric fields in the laboratory system, e, m, and v are the charge, rest mass, and the velocity of the proton, $\gamma = (1 - v^2/c^2)^{1/2}$, $G = (g-2)/2$ (g = gyromagnetic ratio of the proton magnetic moment), and $\mathbf{s}$ is the spin vector from the relation $\mu = g(e/2m)s$ where μ is the magnetic moment of the proton. The computer code which performed the integration of the differential equations of motion (Eq. 3.1) and (Eq. 3.2), as the particles pass through the various magnetic elements of the AtR transfer line, comes under the name RAYTRACE_SPIN and is a modified version of the RAYTRACE computer code[26].

3.2.1 Stable Spin Direction at AGS

The stable spin direction in the AGS is normally along the vertical direction. However, the introduction of the partial Snake makes the stable spin direction depend on the azimuthal location of the beam in the AGS and the beam energy. In particular, at the location of the middle of the C10 straight section which is located 180° from the partial Snake, the directional cosines of the stable spin direction are given by:

$$\begin{aligned} n_{rad} &= 0 \\ n_{vert} &= \frac{1}{N}\sin(\pi G\gamma)\cos(\delta) \\ n_{long} &= \frac{1}{N}\sin(\delta) \end{aligned}$$

where: $N = [1 - \cos^2(\pi G\gamma)\cos^2(\delta)]^{\frac{1}{2}}$ and δ is the spin rotation angle introduced by the Snake, referred to as the Snake strength. When $\delta = 0$ the stable spin direction is along the vertical, independent of the beam energy.

3.2.2 Stable Spin Direction at RHIC Injection Point

With the knowledge of both the stable spin direction and the beam parameters at the middle of the straight section C10 of the AGS, the calculations to obtain the stable spin direction at RHIC injection point proceeded as follows.

a) A given beam energy (corresponding to the AGS closed orbit particle) was chosen.

b) The AtR line was tuned to transport protons with energy equal to the energy of the closed orbit particle in AGS.

c) The closed orbit particle starting from the AGS location C10 was traced (trajectory and spin) down to the H10 extraction section of the AGS, followed by the AtR line to the "RHIC injection point" where the directional cosines of the stable spin direction were recorded.

d) Following the closed orbit particle, step (c) above was repeated for another 999 particles which were randomly selected from the beam ellipsoid of the AGS location C10 and traced down to the "RHIC injection point."

e) From the distribution of the 1000 particles traced, the average value $< S_y >$ and the standard deviation $< (S_y - < S_y >)^2 >^{1/2}$ of the vertical spin direction S_y was calculated at the "RHIC injection point."

The procedure (1) to (5) was repeated for an injected beam with different proton energy.

Stable Spin Direction at RHIC injection point (AGS partial Snake off)

The vertical component S_y of the stable spin direction, at the RHIC injection point, is shown in Fig. 3.4 as a function of $G\gamma$. The empty squares are the S_y components of the central trajectory particle. The filled squares are the average vertical spin projection $< S_y >$, as calculated from the distribution of the S_y component of the 1000 injected particles. The error bars are one standard deviation of the vertical spin projection S_y and contains over 90% of the final spin distribution which is not Gaussian and represent a measure of the spread of the stable spin direction at the RHIC injection point. This spread of the stable spin direction at the RHIC injection point is due to the off central trajectory particles which are experiencing the non vertical fields of the various quadrupoles and combined function magnets (dipole plus quad) of the AtR transfer line. The random sampling of the particles was taken from a beam with a normalized beam emittance of 10 π mm mrad for both vertical and horizontal planes, and one standard deviation of the beam momentum spread $\Delta p/p = \pm 0.05\%$.

The calculations performed to obtain the results shown in Fig. 3.4 assumed that the initial stable spin direction was along the vertical. In a realistic injection of a polarized proton beam the assumption that the spin direction is along the vertical will not be valid, at least at the range of energies covered by the Fig. 3.4. This is because of the partial Snake, the effect of which is discussed next.

Stable Spin Direction at RHIC injection point (AGS partial Snake on)

The vertical component S_y of the stable spin direction at the RHIC injection point when the partial Snake in the AGS is on at a 9° strength, is shown in Fig. 3.5 as a function of the $G\gamma$. The empty squares

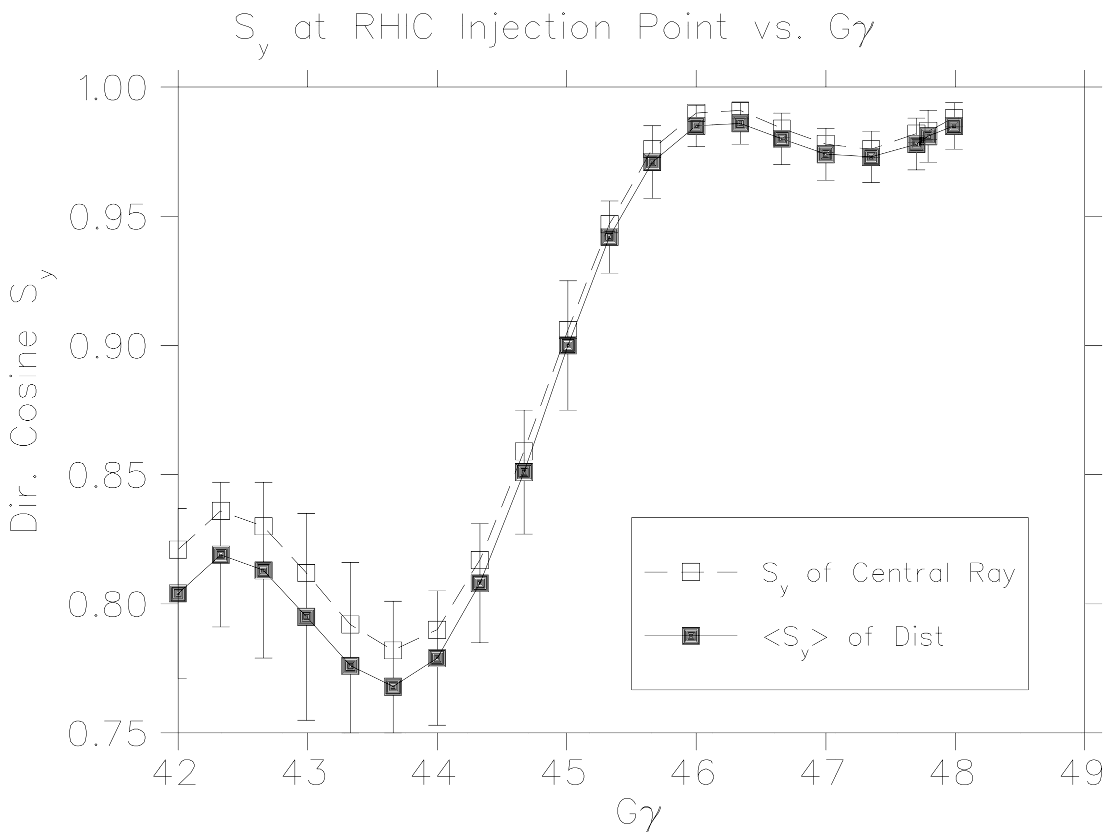

Figure 3.4: The S_y component of the stable spin direction at the RHIC injection point. The empty squares correspond to the central trajectory particle. The filled squares to the average S_y of the spin distribution. The error bars correspond to one standard deviation of the S_y component of the spin distribution. The initial spin direction at AGS has been assumed vertical. (AGS Snake off.)

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are the S_y components of the central trajectory particle. The filled squares are the average vertical spin projection $< S_y >$, as calculated from the distribution of the S_y component of the 1000 particles injected. The error bars are $\pm$ one standard deviation of the vertical spin projection S_y and contain over 90% of the final spin distribution which is not Gaussian. The spread of the stable spin direction at the RHIC injection point as it appears in Fig. 3.5 is due to the off central trajectory particles as mentioned in the previous section. In addition the initial stable spin direction, which is defined by the strength of the partial Snake in the AGS, has a profound effect not only in the spread but also on the average value of the stable spin direction itself at the RHIC injection point. Likewise, as mentioned in the previous section, the random sampling of the particles was taken from a beam with a normalized beam emittance of

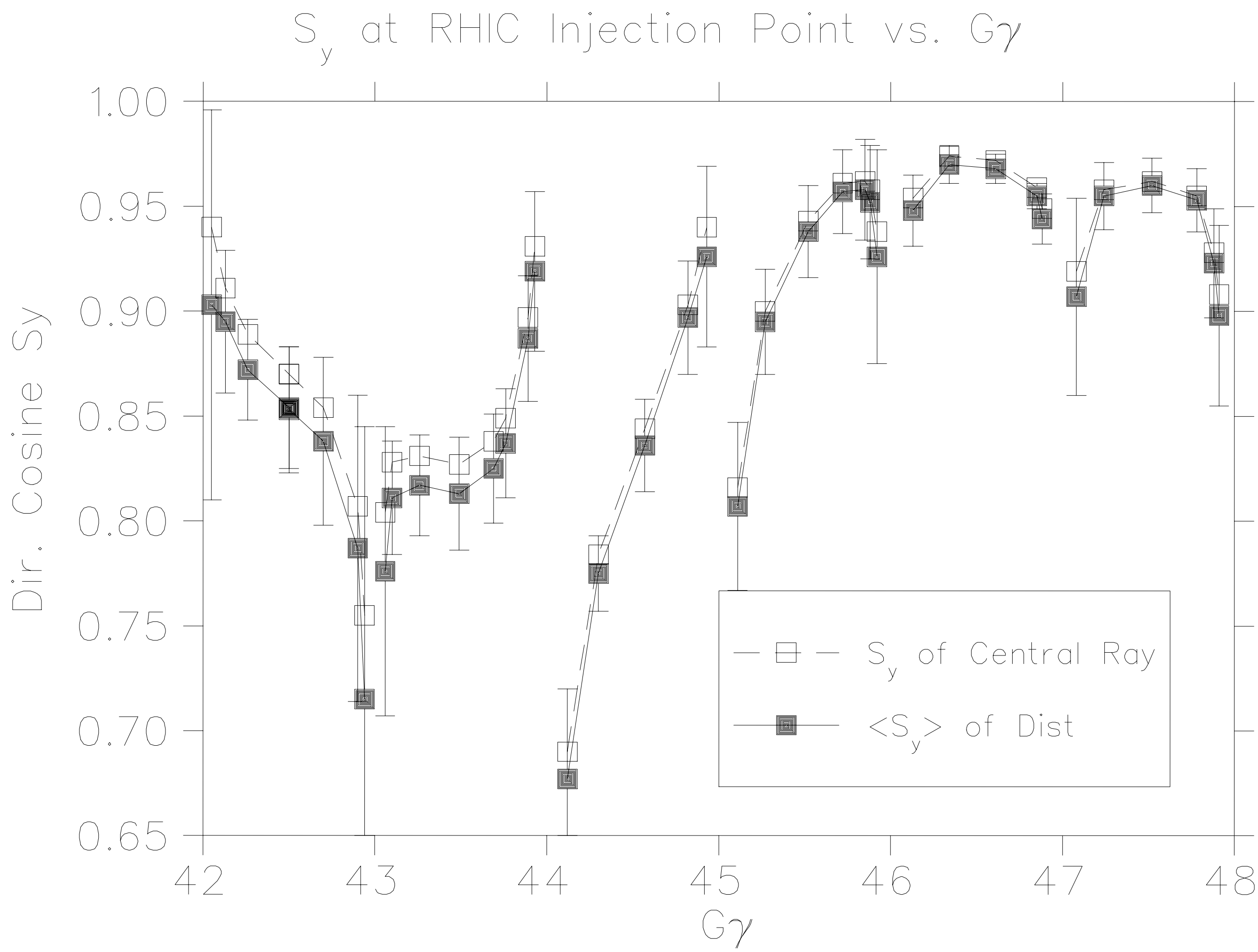

Figure 3.5: The S_y component of the stable spin direction at the RHIC injection point as a function of $G\gamma$. The empty squares correspond to the central trajectory particle. The filled squares are the average S_y of the spin distribution. The error bars correspond to one standard deviation of the S_y component of the spin distribution. The initial spin direction at AGS is defined by the partial Snake in the AGS.

10 π mm mrad for both vertical and horizontal planes, and the beam momentum spread with a standard deviation $\Delta p/p = \pm 0.05\%$.

The stable spin direction of a polarized proton beam circulating in RHIC which will operate with two full Snakes, is along the vertical. Therefore the stable spin direction of the injected polarized beam should be along the vertical if possible. The results in Fig. 3.5 will serve as a guide to help choose the injection energy which will provide the maximum vertical components of the stable spin direction with minimum spread. At injection proton beam energies corresponding to $G\gamma = integer$ the stable spin direction at AGS is not well defined and neither is the stable spin direction at the RHIC injection point. (See Fig. 3.5.)

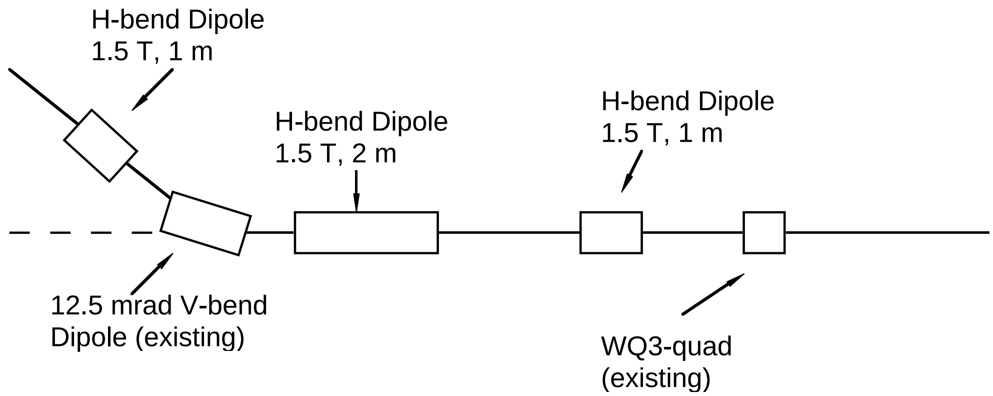

Figure 3.6: Schematic diagram of the section of the AtR transfer line with the three horizontally bending magnets which can optimize the AtR polarization transfer at various proton injection energies.

3.2.3 Optimization of Stable Spin Direction with Energy

The preceding two sections suggest that the optimum energy range for injection of a polarized proton beam into RHIC corresponds to values of $G\gamma$ from $\sim$45 to 48. However it is possible to alter the energy range for optimum polarization transfer of an injected beam into RHIC by employing three additional horizontally bending dipole magnets in the AtR transfer line. A schematic diagram of the the section of the AtR transfer line which includes the three added dipoles, is shown in Fig. 3.6.

The first of the three dipoles is placed in front of the second 12.5 mrad vertically bending magnet of the 20° bend of the ATR line (see Fig. 3.6). The second dipole shown in Fig. 3.6 is to be placed just after the second 12.5 mrad vertically bending magnet with the center of the third dipole placed downstream from the center of the second dipole, at a distance equal to the distance of the centers of the first two horizontally bending dipoles. This dipole arrangement is not dispersive, and will only affect the stable spin direction after the vertical magnet shown in Fig. 3.6. The beam trajectory will be restored at the entrance of the WQ3 quadrupole (Fig. 3.6) and the beam parameters will be practically the same as before the inclusion of the dipoles. The next two sections present and discuss the results of the stable spin direction at the RHIC injection point when the three horizontal dipoles shown in Fig. 3.6 are excited to 1.5 Tesla each in a direction such that the first dipole will bend the beam to the right, the second to the left, and the third one to the right.

Stable Spin Direction (AGS partial Snake off; AtR modified)

With the AtR transfer line modified by the insertion of the three dipoles mentioned in the previous subsection, the calculation for the stable spin direction at the RHIC injection point was repeated. The

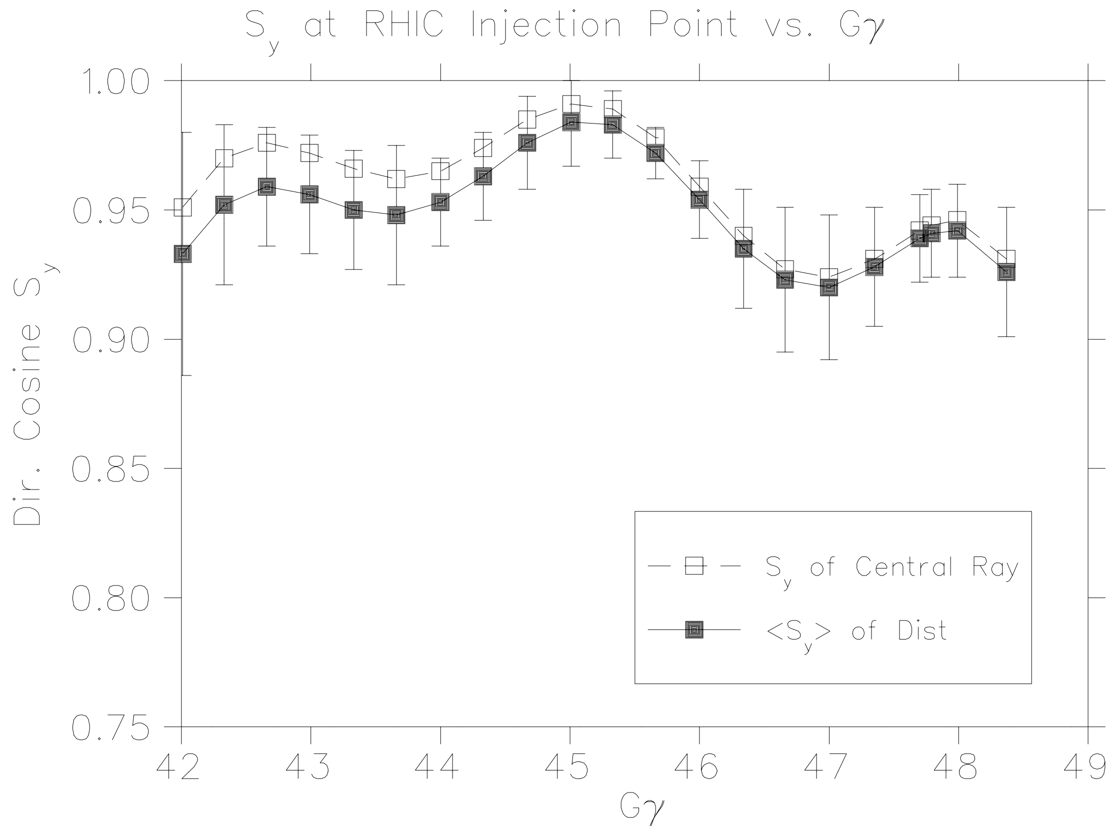

Figure 3.7: The S_y component of the stable spin direction at the RHIC injection point. The empty squares correspond to the central trajectory particle. The filled squares to the average S_y of the spin distribution. The error bars correspond to one standard deviation of the S_y component of the spin distribution. The initial spin direction in the AGS has been assumed vertical. (Partial Snake is off.) The three dipoles of Fig. 3.6 are excited to 1.5 T each.

vertical component S_y of the stable spin direction at the RHIC injection point is shown in Fig. 3.7 as a function of $G\gamma$.

Comparing Fig. 3.4 with Fig. 3.7, the shift of the optimum stable spin direction to lower energies is obvious. The assumption made in the calculation to obtain the results shown in Fig. 3.4 are also valid in obtaining the results of Fig. 3.7

Stable Spin Direction (AGS partial Snake on; AtR line modified)

In these calculations the AtR transfer line is modified as mentioned earlier, and the AGS partial Snake is turned on at a 9° strength. The vertical component S_y of the stable spin direction at the RHIC injection

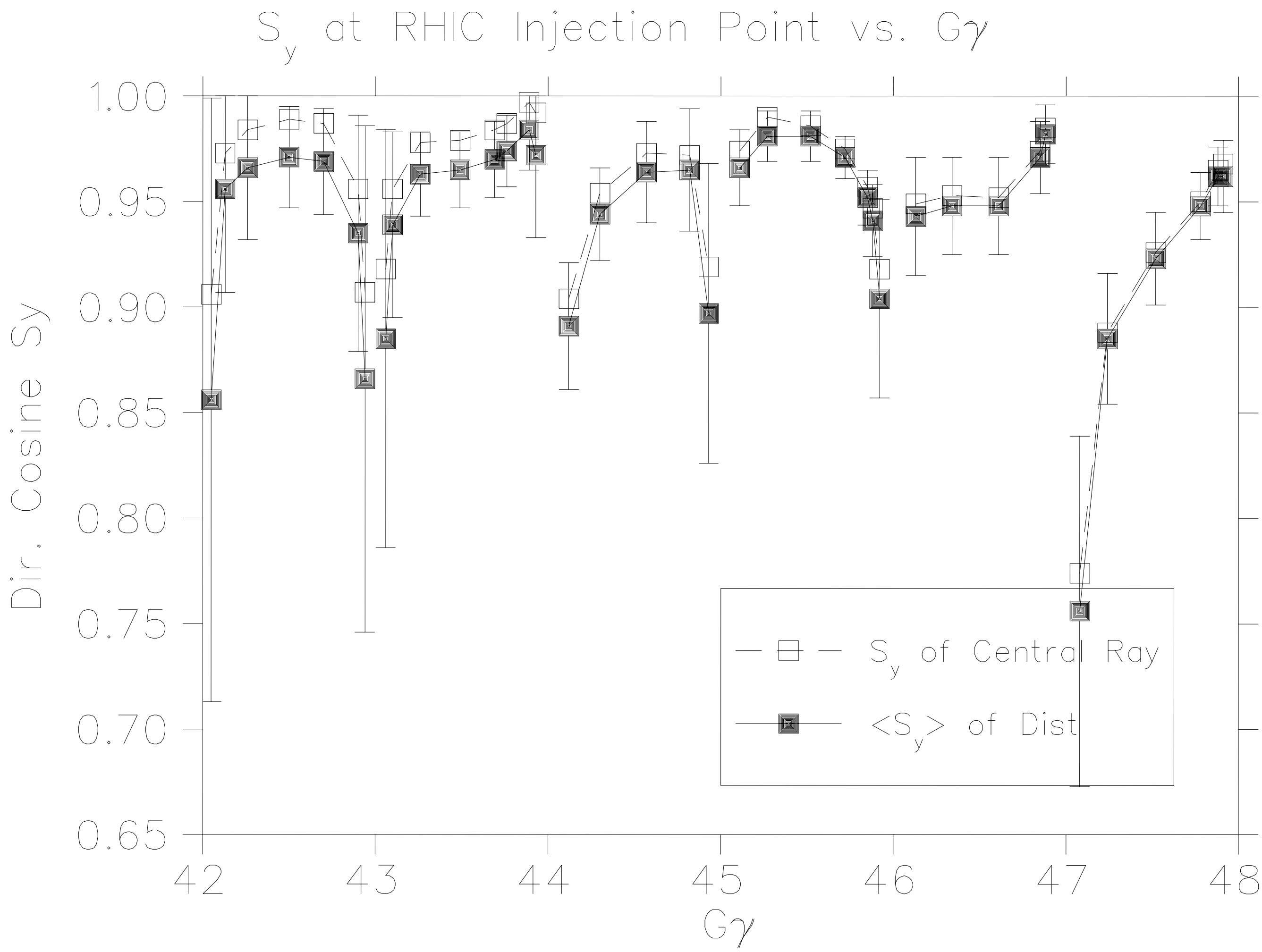

Figure 3.8: The S_y component of the stable spin direction at the RHIC injection point as a function of $G\gamma$. The empty squares correspond to the central trajectory particle. The filled squares is the average S_y of the spin distribution. The error bars correspond to one standard deviation of the S_y component of the spin distribution. The three dipoles of Fig. 3.6 are excited to 1.5 T each. The initial spin direction at AGS is defined by the partial Snake in AGS.

point when the partial Snake in AGS is on, is shown in Fig. 3.8 as a function of $G\gamma$.

Comparing Fig. 3.5 with Fig. 3.8 the shift of the optimum stable spin direction to lower energies is again obvious.

3.2.4 Conclusions

The stable spin direction of a polarized proton beam at the RHIC injection point, has been studied as a function of proton energy. It is shown that the stable spin direction at the RHIC injection point depends upon the injected proton energy. Optimum polarization transfer can be defined as the condition where the average stable spin direction of the protons at the RHIC injection point is "almost" vertical with the minimum possible spread among the particles of the bunch.